ON THE THEORY OF THE GYRO HORIZON COMPASS

(K TEORII GIROGORIZONTKOMPASA)

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Iu.K. ZHBANOV
(Moscow)

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The present article studies the effect of arbitrary misalignments and imbalances on the behavior of a gyro horizon compass. It gives an estimate of the azimuthal deviations of an unbalanced compass in a maneuvering vehicle and indicates the possibility of accurately measuring the imbalance from the curve of its azimuthal oscillations on a fixed base if the damping is disconnected.

An ideal gyro horizon compass [1] is a two-gyroscope pendulum with specially selected parameters and couplings. A variant of such a pendulum, constructed in the form of a floating gyrosphere, is schematically illustrated in Fig. 1. $\mathbf{H}$ denotes the total angular momentum of the two gyroscopes; $\mathbf{r}$ is the unit vector along the compass vertical, directed from the center of gravity of the gyrosphere toward its geometrical center, which acts as a point of support; a is the distance from the center of gravity to the point of support (metacentric height); m is the mass of the gyrosphere. We shall call the trihedron

$$\left\{\frac{\mathbf{H}}{H},\ \mathbf{r},\ \frac{\mathbf{H}\times\mathbf{r}}{H}\right\}$$

which defines the orientation of the instrument, the "compass trihedron" The position and maneuvering of the point of support (or the base) with respect to the surface of the earth will be given by the unit vector along the local vertical $\mathbf{r}_1(t)$ directed from the center of the earth to the point of support.

The special choice of the parameters of the gyropendulum reduces to the fact that for any H in the allowable range two conditions are satisfied:

1) The angular momentum vector is perpendicular to the compass vertical

$$\mathbf{H}\cdot\mathbf{r}=0$$

2) The angular velocity of the compass trihedron in inertial space with respect to the total angular momentum is determined only by the internal couplings and is expressed by the formula

$$\omega_0 = \frac{\mathbf{H}}{amR} \qquad (R \text{ is the radius of the earth})$$

1. The system of equations which describes the behavior of an ideal compass is of the form

$$\dot{\mathbf{H}} = amR\,\mathbf{r}\times\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right), \quad \dot{\mathbf{r}} = \boldsymbol{\omega}\times\mathbf{r}, \quad \boldsymbol{\omega} = \frac{\mathbf{H}\times\dot{\mathbf{H}}}{H^2} + \frac{\mathbf{H}}{amR} \tag{1.1}$$

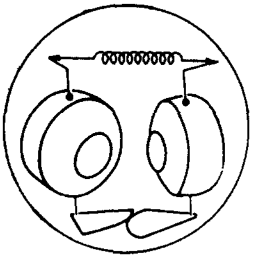

Fig. 1.

The dots indicate differentiation in inertial space. The right-hand side of the first equation represents the torque with respect to the point of support (Fig. 2) of the total force $(-mg\mathbf{r}_1 - mR\ddot{\mathbf{r}}_1)$ acting with a lever arm $-a\mathbf{r}$. The angular velocity vector of rotation of the compass trihedron, denoted by $\boldsymbol{\omega}$, is determined to within the component along $\mathbf{H}$ by the motion of $\mathbf{H}$ itself, and the magnitude of the component along $\mathbf{H}$ is determined by condition (2); we shall consider below the question of satisfying this condition. Eliminating $\boldsymbol{\omega}$ from the system (1.1), we obtain

$$\dot{\mathbf{H}} = amR\,\mathbf{r}\times\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right), \qquad \dot{\mathbf{r}} = \frac{\mathbf{H}\times\mathbf{r}}{amR} \tag{1.2}$$

By the second equation of (1.2), we have

$$\mathbf{H} = amR\,\mathbf{r}\times\dot{\mathbf{r}} \tag{1.3}$$

After eliminating $\mathbf{H}$ on the assumption that amR = const, the system becomes [2]

$$\mathbf{r}\times\ddot{\mathbf{r}} = \mathbf{r}\times\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right)$$

or

$$\ddot{\mathbf{r}} + \alpha\mathbf{r} = \ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1 \tag{1.4}$$

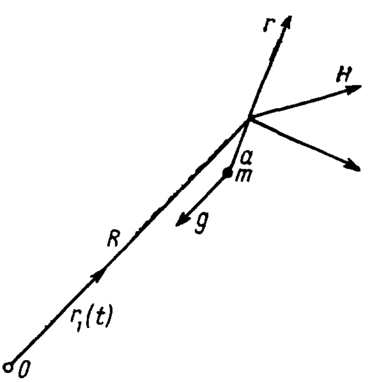

Fig. 2.

The quantity α in (1.4) is characteristic of the reaction of the coupling r = 1. If the initial conditions are

$$\mathbf{r}(0) = \mathbf{r}_1(0), \qquad \dot{\mathbf{r}}(0) = \dot{\mathbf{r}}_1(0) \tag{1.5}$$

the particular solution of the system (1.4) will be

$$\mathbf{r} = \mathbf{r}_1(t) \tag{1.6}$$

The orientation of the compass trihedron in this case is determined only by the position and velocity of the base, which makes possible the use of the device as a navigational instrument.

It should be noted that a compass with an arbitrarily variable metacentric height $a(t)$ can have all the properties of an ideal compass if we apply to it the additional moment

$$\mathbf{M}^* = \frac{\dot{a}}{a}\mathbf{H}$$

leaving conditions (1) and (2) valid. The above transformation will lead to the equations (1.4) in this case as well.

2. The possibility and the methods of realizing the condition (1) are obvious. Let us consider the condition (2). The total angular momentum $\mathbf{H}$ is obtained from two gyroscopes with individual angular momenta $\mathbf{B}_1$ and $\mathbf{B}_2$. The interaction of the gyroscopes reduces to the application of a torque $\mathbf{N}_1$ to the first gyroscope and a torque $\mathbf{N}_2$ to the second gyroscope. The torques $\mathbf{N}_1$ and $\mathbf{N}_2$ are equal in magnitude, opposite in sense and perpendicular to both angular momenta $\mathbf{B}_1$ and $\mathbf{B}_2$, so that we may write

$$\mathbf{N}_1 = -N\frac{\mathbf{B}_1 \times \mathbf{B}_2}{|\mathbf{B}_1 \times \mathbf{B}_2|}, \qquad \mathbf{N}_2 = N\frac{\mathbf{B}_1 \times \mathbf{B}_2}{|\mathbf{B}_1 \times \mathbf{B}_2|} \tag{2.1}$$

The action of $\mathbf{N}_1$ and $\mathbf{N}_2$ produces a rotation of the plane of the angular momenta B_1B_2 with respect to inertial space, with an angular velocity $\boldsymbol{\omega}_1$ such that

$$\boldsymbol{\omega}_1 \times \mathbf{B}_1 = \mathbf{N}_1, \qquad \boldsymbol{\omega}_1 \times \mathbf{B}_2 = \mathbf{N}_2 \tag{2.2}$$

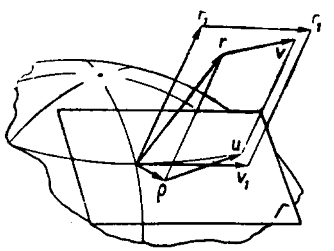

Fig. 3.

From (2.2), in view of (2.1), we can find

$$\boldsymbol{\omega}_1 = \frac{N}{|\mathbf{B}_1 \times \mathbf{B}_2|}(\mathbf{B}_1 + \mathbf{B}_2) = \frac{N}{|\mathbf{B}_1 \times \mathbf{B}_2|}\mathbf{H} \tag{2.3}$$

If the plane of the angular momenta does does not change its position in the compass trihedron, then $\boldsymbol{\omega}_0 = \boldsymbol{\omega}_1$, and condition (2) reduces to the requirement [1]

$$N = \frac{|\mathbf{B}_1 \times \mathbf{B}_2|}{amR}, \qquad \text{or} \qquad N = \frac{B_1B_2}{amR}\sin 2\varepsilon \tag{2.4}$$

It should be noted that if V, the potential energy of the system, changes with the change in the angle 2ε between the gyroscopes, then $N(\varepsilon) = -\partial V/\partial(2\varepsilon)$, and by (2.3)

$$\boldsymbol{\omega}_1 = -\frac{\partial V}{\partial(2\varepsilon)}\frac{\mathbf{H}}{|\mathbf{B}_1 \times \mathbf{B}_2|} = \frac{\partial V}{\partial H}\frac{\mathbf{H}}{H} \tag{2.5}$$

Formula (2.5) makes it possible to take into account both the action of the internal elastic coupling and the effect of the external forces.

In accordance with (2.4), condition (2) requires only the introduction of the elastic coupling; however, in order to find the compass north, we introduce an additional sector or anti-parallelogram coupling between the gyroscopes. In this event the axis of the case, which is completely determined, is always the bisecting line between the angular momenta and may be taken as compass north. The necessity for sector coupling will be eliminated if we can maintain conditions (1) and (2) and find the direction of $\mathbf{H}$ by another method.

3. In order to transform the compass equations, we introduce a new variable, the vector

$$\mathbf{v} = \frac{\mathbf{H} \times \mathbf{r}}{amR} \tag{3.1}$$

directed along the east axis of the compass. Using (3.1) to eliminate $\mathbf{H}$ from the system (1.2) and using the coupling equation $r = 1$ to calculate its reaction, we obtain

$$\dot{\mathbf{v}} = \ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1 - \mathbf{r}\left[v^2 - \mathbf{r}\cdot\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right)\right], \qquad \dot{\mathbf{r}} = \mathbf{v} \tag{3.2}$$

The system (3.2) is projected onto a horizontal plane Γ perpendicular to $\mathbf{r}_1$ (Fig. 3). If we designate the projections of the vectors $\mathbf{r}$, $\mathbf{v}$, $\dot{\mathbf{r}}_1$ onto the plane Γ by ρ, $\mathbf{u}$, $\mathbf{v}_1$, then

$$\mathbf{r} = \rho + k\mathbf{r}_1, \qquad \mathbf{v} = \mathbf{u} + \theta\mathbf{r}_1, \qquad \dot{\mathbf{r}}_1 = \mathbf{v}_1 \qquad \left(k = \sqrt{1-\rho^2},\ \theta = \frac{\mathbf{u}\cdot\rho}{k}\right) \tag{3.3}$$

In the plane Γ we introduce a coordinate system which does not rotate about $\mathbf{r}_1$. If differentiation in this system is denoted by d/dt, then

$$\dot{\mathbf{r}} = \frac{d\rho}{dt} + k\mathbf{v}_1 + \mathbf{r}_1(\dot{k} - \mathbf{v}_1\cdot\rho), \quad \ddot{\mathbf{r}}_1 = \frac{d\mathbf{v}_1}{dt} - \mathbf{r}_1 v_1^2, \qquad \dot{\mathbf{v}} = \frac{d\mathbf{u}}{dt} + \theta\mathbf{v}_1 + \mathbf{r}_1(\dot{\theta} - \mathbf{v}_1\cdot\mathbf{u}) \tag{3.4}$$

Substituting (3.3) and (3.4) into (3.2), we obtain

$$\begin{gathered}\frac{d\rho}{dt} + k\mathbf{v}_1 + \mathbf{r}_1(\dot{k} - \mathbf{v}_1\cdot\rho) = \mathbf{u} + \theta\mathbf{r}_1 \\ \frac{d\mathbf{u}}{dt} + \theta\mathbf{v}_1 + \mathbf{r}_1(\dot{\theta} - \mathbf{v}_1\cdot\mathbf{u}) = \frac{d\mathbf{v}_1}{dt} - \mathbf{r}_1 v_1^2 + \frac{g}{R}\mathbf{r}_1 - \\ -(\rho + k\mathbf{r}_1)\left[k\frac{g}{R} + \rho\cdot\frac{d\mathbf{v}_1}{dt} - k_1 v_1^2 + v^2\right]\end{gathered} \tag{3.5}$$

The projection onto the plane Γ is

$$\frac{d\rho}{dt} + k\mathbf{v}_1 = \mathbf{u}, \quad \frac{d\mathbf{u}}{dt} + \theta\mathbf{v}_1 = \frac{d\mathbf{v}_1}{dt} - \rho\left[k\frac{g}{R} + \rho\cdot\frac{d\mathbf{v}_1}{dt} - kv_1^2 + v^2\right] \tag{3.6}$$

The vector ρ gives a direct indication of the deviation of compass vertical from true vertical. The angle between the vectors $\mathbf{u}$ and $\mathbf{v}_1$ indicates the azimuthal rotation of the compass trihedron away from its undisturbed position. If we neglect the vertical component of the earth's centrifugal force $\theta \mathbf{v}_1$, then the linear part of the system (3.6) will be

$$\frac{d\rho}{dt} = \mathbf{u} - \mathbf{v}_1, \qquad \frac{d\mathbf{u}}{dt} - \frac{d\mathbf{v}_1}{dt} = -\frac{g}{R}\rho \tag{3.7}$$

Eliminating ρ and setting $\mathbf{u} - \mathbf{v}_1 = \mathbf{w}$, we obtain from (3.7) [3]

$$\frac{d^2\mathbf{w}}{dt^2} + \frac{g}{R}\mathbf{w} = 0 \tag{3.8}$$

In accordance with (3.8), the vector $\mathbf{w}$ will, in the general case, describe an ellipse with a Schuler frequency, at $\sqrt{(g/R}$ rad/sec. The azimuthal deviation, being the angle between the vectors $\mathbf{v}_1$ and $\mathbf{v}_1 + \mathbf{w}$, depends on $\mathbf{v}_1$, which is determined by the location and maneuver of the base. In the stationary position, the vector $\mathbf{v}_1$ rotates in the plane Γ with a velocity $U \sin \varphi$ (the vertical component of angular velocity of rotation of the earth), so that in the azimuthal oscillations of the compass we observe pulsations with a period of the order of $2\pi/U \sin \varphi$. The amplitude of the azimuthal oscillations is maximum (antinode of the pulsations), when the vector $\mathbf{v}_1$ is perpendicular to the major axis of the ellipse of the oscillations of $\mathbf{w}$, and it is a minimum (node) when $\mathbf{v}_1$ is directed along that axis.

4. The action of any oscillations can be reduced, in this or another manner, to some external $\mathbf{M}^*$ and to a distortion of the angular velocity of the compass trihedron with respect to the angular momentum, Δ. The torque $\mathbf{M}^*$ may be represented in the form

$$\mathbf{M}^* = amR\,(\mathbf{r} \times \mathbf{F} + \mathbf{r}M_z) \qquad (\mathbf{r} \cdot \mathbf{F} = 0)$$

The compass equation system, similar to (1.1), becomes

$$\dot{\mathbf{H}} = amR\,\mathbf{r} \times \left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right) + amR\,\mathbf{r} \times \mathbf{F} + \mathbf{r}\,amR\,M_z$$

$$\dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}, \qquad \boldsymbol{\omega} = \frac{\mathbf{H} \times \dot{\mathbf{H}}}{H^2} + \frac{\mathbf{H}}{amR} + \frac{\mathbf{H}}{H}\Delta \tag{4.1}$$

After eliminating ω and introducing the variable $\mathbf{v}$ in accordance with formula (3.1), we obtain a system similar to (3.2)

$$\dot{\mathbf{v}} = \ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1 + \mathbf{F} - \mathbf{r}\left[\left(1 + \frac{\Delta}{v}\right)v^2 + \mathbf{r} \cdot \left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1 + \mathbf{F}\right)\right]$$

$$\dot{\mathbf{r}} = \left(1 + \frac{\Delta}{v}\right)\mathbf{v} - M_z\frac{\mathbf{r} \times \mathbf{v}}{v^2} \tag{4.2}$$

Using equations (3.3) and (3.4) directly, we can write the projection of the system (4.2) on the plane Γ

$$\frac{d\rho}{dt}=\mathbf{u}-k\mathbf{v}_1+\frac{\Delta}{v}\mathbf{u}-\frac{M_z}{v^2}\mathbf{r}_1\times(k\mathbf{u}+\theta\rho)$$
$$\frac{d\mathbf{u}}{dt}=\frac{d\mathbf{v}_1}{dt}-\rho\left[k\frac{g}{R}-kv_1^2+\left(1+\frac{\Delta}{v}\right)v^2+\rho\cdot\left(\frac{d\mathbf{v}_1}{dt}+\mathbf{F}\right)\right]+\mathbf{F}-\theta\mathbf{v}_1 \qquad (4.3)$$

If we consider that the disturbance Δ is of the order of v and if we neglect the vertical component of the centrifugal force of the earth, then after dropping terms known to be second-order terms, the system (4.3) becomes

$$\frac{d\rho}{dt}=\mathbf{u}-\mathbf{v}_1+\frac{\mathbf{u}}{u}\,\Delta-\frac{M_z}{u}\frac{\mathbf{r}_1\times\mathbf{u}}{u}, \qquad \frac{d\mathbf{u}}{dt}=\frac{d\mathbf{v}_1}{dt}-\frac{g}{R}\rho+\mathbf{F} \qquad (4.4)$$

5. We consider a compass (Fig. 4) in which the rotation axes of the cases are not parallel to one another and are arbitrarily oriented with respect to the lines of "pendulosity", the center of gravity of each gyroscope is displaced from the axis of rotation of the case, and the angular momenta of the gyroscopes are not perpendicular to the rotation axis of their cases. In this case there is no simple relation between the directions of the compass vertical $\mathbf{r}$ and the angular momenta $\mathbf{B}_1$ and $\mathbf{B}_2$, so that it is advisable to consider two trihedra: the base trihedron

$$\left\{\frac{\mathbf{B}_1+\mathbf{B}_2}{|\mathbf{B}_1+\mathbf{B}_2|},\ \frac{\mathbf{B}_1\times\mathbf{B}_2}{|\mathbf{B}_1\times\mathbf{B}_2|},\ \frac{(\mathbf{B}_1+\mathbf{B}_2)\times(\mathbf{B}_1\times\mathbf{B}_2)}{|\mathbf{B}_1+\mathbf{B}_2|\,|\mathbf{B}_1\times\mathbf{B}_2|}\right\}$$

and the compass trihedron

$$\left\{\frac{\mathbf{H}}{H},\ \mathbf{r},\ \frac{\mathbf{H}\times\mathbf{r}}{H}\right\}$$

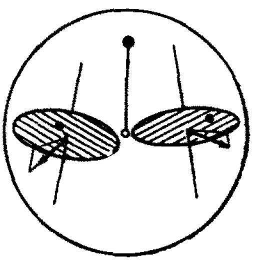

Fig. 4.

where $\mathbf{H}$ is the component of the vector $\mathbf{B}_1+\mathbf{B}_2$ perpendicular to $\mathbf{r}$. We introduce the notation $\mathbf{H}_1=\mathbf{B}_1+\mathbf{B}_2$; we than have

$$\mathbf{H}=\mathbf{r}\times(\mathbf{H}_1\times\mathbf{r}), \qquad \mathbf{H}_1=\mathbf{H}+\mathbf{r}(\mathbf{H}_1\cdot\mathbf{r})$$

As a result of the misalignments and imbalances, the orientation of the compass vertical $\mathbf{r}$ and the position of the center of gravity $-a\mathbf{r}$ with respect to the base trihedron, are found to be dependent on the angle of separation of the gyroscopes, that is, on H_1. As H_1 varies, the vector $\mathbf{r}$ is displaced in the base trihedron with a velocity $(\partial\mathbf{r}/\partial H_1)(dH_1/dt)$, so that the compass equations, similar to (1.1), become

$$\dot{\mathbf{H}}_1=amR\,\mathbf{r}\times\left(\ddot{\mathbf{r}}_1+\frac{g}{R}\mathbf{r}_1\right), \qquad \dot{\mathbf{r}}=\omega_1\times\mathbf{r}+\frac{\partial\mathbf{r}}{\partial H_1}\frac{dH_1}{dt}$$
$$\omega_1=\frac{\mathbf{H}_1\times\dot{\mathbf{H}}_1}{H_1^2}+\frac{\mathbf{H}_1}{amR}-\frac{\mathbf{H}_1}{H_1}mR\left(\ddot{\mathbf{r}}_1+\frac{g}{R}\mathbf{r}_1\right)\cdot\frac{\partial(a\mathbf{r})}{\partial H_1} \qquad (5.1)$$

Here ω_1 is the angular velocity vector of the rotation of the base trihedron. The last term on the right-hand side of the third equation is due to the fact that as H_1 varies, work is done not only by the internal forces (see Section 2) but also by the external force $(-m\ddot{R}\mathbf{r}_1 - mg\mathbf{r}_1)$ on the displacement of the center of gravity $(-\partial(a\mathbf{r})/\partial H_1)(dH_1/dt)$. Passing to vectors of the compass trihedron in (5.1), we obtain

$$\dot{\mathbf{H}} = amR\,\mathbf{r}\times\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right) - \frac{d}{dt}[\mathbf{r}(\mathbf{H}_1\cdot\mathbf{r})],\ \dot{\mathbf{r}} = \boldsymbol{\omega}\times\mathbf{r}, \qquad \boldsymbol{\omega} = \frac{\mathbf{H}\times\dot{\mathbf{H}}}{H^2} + \omega_H\frac{\mathbf{H}}{H} \tag{5.2}$$

The expression for the projection of the angular velocity of the compass trihedron on the direction of the angular momentum is obtained, using (5.1), by the formula

$$\omega_H = (\mathbf{r}\times\dot{\mathbf{r}})\cdot\frac{\mathbf{H}}{H} \tag{5.3}$$

Carrying out the calculations, we obtain

$$\omega_H = \frac{H}{amR} - \frac{H}{H_1}mR\frac{\partial a}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right)\cdot\mathbf{r} - \frac{amR}{H_1}\frac{\partial(\mathbf{H}_1\cdot\mathbf{r})}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right)\cdot\frac{\mathbf{H}}{H} \tag{5.4}$$

When we have set up (5.4) and transformed the right-hand side of the first equation, the system (5.2) reduces to a form similar to (4.1)

$$\dot{\mathbf{H}} = amR\,\mathbf{r}\times\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right) - \mathbf{r}\frac{amR}{H_1}\frac{\partial(\dot{\mathbf{H}}_1\cdot\mathbf{r})}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right)\cdot(\mathbf{H}\times\mathbf{r}) + \mathbf{r}\times\boldsymbol{\omega}(\mathbf{H}_1\cdot\mathbf{r})$$

$$\dot{\mathbf{r}} = \boldsymbol{\omega}\times\mathbf{r} \tag{5.5}$$

$$\boldsymbol{\omega} = \frac{\mathbf{H}\times\dot{\mathbf{H}}}{H^2} + \frac{\mathbf{H}}{amR} - \frac{\mathbf{H}}{H_1}mR\frac{\partial a}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{g\mathbf{r}_1}{R}\right)\cdot\mathbf{r} - \frac{\mathbf{H}}{H}\frac{amR}{H_1}\frac{\partial(\mathbf{H}_1\cdot\mathbf{r})}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{g\mathbf{r}_1}{R}\right)\cdot\frac{\mathbf{H}}{H}$$

It is easily observed that the disturbances, which distinguish the system (5.5) from (1.1), vanish if the conditions

$$a = \text{const}, \qquad \mathbf{H}_1\cdot\mathbf{r} = 0 \tag{5.6}$$

are satisfied.

In accordance with (5.6), the east imbalance is constant, and furthermore, the deviation of the axes of rotation of the cases about the east axis by equal angles in different directions, resulting in a variable east imbalance, do not produce any disturbances.

6. If we desire an approximate formulation, then, comparing (5.5) with (4.1), we can consider

$$\Delta = -\frac{amR}{H_1}\frac{\partial(\mathbf{H}_1\cdot\mathbf{r})}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{\partial}{R}\mathbf{r}_1\right)\cdot\frac{\mathbf{H}}{H}, \qquad \mathbf{F} = 0$$

$$M_z = -\frac{1}{H_1}\frac{\partial(\mathbf{H}_1\cdot\mathbf{r})}{\partial H_1}\left(\ddot{\mathbf{r}}_1 + \frac{g}{R}\mathbf{r}_1\right)\cdot(\mathbf{H}\times\mathbf{r}) \tag{6.1}$$

Passing from (6.1) to the variable system (4.4) and retaining the linear term, we obtain

$$\Delta = -\delta \frac{1}{u}\left(\frac{d\mathbf{v}_1}{dt} - \frac{g}{R}\rho\right) \cdot \frac{\mathbf{r}_1 \times \mathbf{u}}{u}, \qquad \mathbf{F} = 0, \qquad M_z = -\delta\left(\frac{d\mathbf{v}_1}{dt} - \frac{g}{R}\rho\right) \cdot \frac{\mathbf{u}}{u} \tag{6.2}$$

Here we introduce the notation for the generalized imbalance angle

$$\delta = \frac{\partial (\mathbf{H}_1 \quad \mathbf{r})}{\partial H_1} \tag{6.3}$$

Taking (6.2) into consideration, the system (4.4) becomes

$$\frac{d\rho}{dt} = \mathbf{u} - \mathbf{v}_1 + \delta \frac{1}{u}\mathbf{r}_1 \times \frac{d\mathbf{u}}{dt}, \qquad \frac{d\mathbf{u}}{dt} = \frac{d\mathbf{v}_1}{dt} - \frac{g}{R}\rho \tag{6.4}$$

Eliminating ρ and using the variable $\mathbf{w}$ (3.8), we obtain

$$\frac{d^2\mathbf{w}}{dt^2} + \frac{g}{R}\mathbf{w} + \frac{\delta}{u}\mathbf{r}_1 \times \frac{d\mathbf{w}}{dt} = -\frac{\delta}{u}\mathbf{r}_1 \times \frac{d\mathbf{v}_1}{dt} \tag{6.5}$$

According to (6.5), for the case of free oscillations ($d\mathbf{v}_1/dt \approx 0$) the vector $\mathbf{w}$ describes an ellipse rotating in the plane Γ with an angular velocity of

$$\Omega = -\frac{\delta}{2u}\frac{g}{R}$$

The frequency of the pulsations of the azimuthal oscillations is equal to the difference between the angular velocities of the geographical axes and the axes of the ellipse, which corresponds to a pulsation period of

$$T = \frac{2\pi}{\left| U \sin\varphi + \frac{\delta}{2u}\frac{g}{R} \right|}, \text{ or in hours } \quad T = \frac{24}{\left| \sin\varphi + \frac{144}{\cos\varphi}\delta \right|} \tag{6.6}$$

After a short maneuver which changes the absolute velocity $\mathbf{v}_1$ by $\Delta\mathbf{v}$, the state of a previously undisturbed compass is given, according to (6.4), by the following values of the coordinates

$$\rho = \frac{\delta}{u}\mathbf{r}_1 \times \Delta\mathbf{v}, \qquad \mathbf{u} = \mathbf{v}_1$$

The corresponding azimuthal deviation at the antinode of the pulsations is expressed by the formula

$$\Delta\alpha_m = \frac{17}{\cos\varphi}\,\delta\,\frac{\Delta v}{v_1} \tag{6.7}$$

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